

Test

1. Fie functia $f: \mathbb{R} \rightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right), f(x) = \arctg x$

a) Calculati $\int_0^1 \frac{1}{x^2+1} f(x) dx$

b) Calculati $\int_0^1 f(x) dx$

c) Calculati $\int_{-1}^1 f^{101}(x) dx$

2. Se considera sirul $(I_n)_{n \geq 0}$ definit prin $I_n = \int_{\frac{n+1}{e^{\frac{n+1}{2}}}}^{\frac{n+2}{e^{\frac{n+2}{2}}}} \frac{2 \ln x - 1}{x} dx$.

a) Calculati I_0 .

b) Sa se arate ca sirul $(I_n)_{n \geq 0}$ este o progresie aritmetica, precizand ratia ei.

3. Fie sirul $I_n = \int_0^{\frac{\pi}{4}} (tg x)^n dx, n \in \mathbb{N}, n \geq 2$

a) Calculati I_2 .

b) Aratati ca $I_n + I_{n+2} = \frac{1}{n+1}, n \geq 2$.

4. Fie $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = \int_0^{x^2} (t^3 - 3t + 2) e^t dt$.

a) calculati $f(1)$.

b) rezolvati ecuatia $f'(x) = 0$.